World 8-1 Cubes and Cube Roots

1) Write the square or cube of each number

a) $4^2 = 4x4 = 16$ $m^2 =$ _____ $3^3 =$ _____

b) $6^3 =$ _____ $5^2 =$ _____ $(-3)^2 =$ _____

c) $10^3 =$ _____ $13^2 =$ _____ $q^3 =$ _____

d) $24^3 =$ _____ $(-5)^2 =$ _____ $21^3 =$ _____

e) $11^2 =$ _____ $43^3 =$ _____ $i^3 =$ _____

2) Write the square root

a) $36 = 6^2$ $64 =$ ____ $81 =$ ____ $196 =$ ____ $324 =$ ____ $2500 =$ ___

b) $400 =$ ___ $49 =$ ____ $225 =$ ____ $121 =$ ____ $144 =$ ____ $900 =$ ___

c) $16 =$ ___ $25 =$ ____ $a^2 =$ ____ $b^4 =$ ____ $625 =$ ____ $4 =$ ___

3) Write the cube root

a) $216 = 6^3$ $8 =$ ____ $1000 =$ ____ $64 =$ ____ $100 =$ ____ $125 =$ ___

b) $1728 =$ ___ $49 =$ ____ $-729 =$ ____ $512 =$ ____ $343 =$ ____ $6859 =$ ___

4) Calculate the following cube or cube roots roots

a) 3^3 _____ e) $\sqrt[3]{8}$ _____ i) 7^3 _____ m) $\sqrt[3]{(3+5)}$ _____ q) $\sqrt[3]{27}$ _____

b) $\sqrt[3]{64}$ _____ f) $\sqrt[3]{1}$ _____ j) 10^3 _____ n) $\sqrt[3]{3^3-27}$ _____ r) $\sqrt[3]{-64}$ _____

c) $\sqrt[3]{1728}$ _____ g) $\sqrt[3]{343}$ _____ k) $\sqrt[3]{-1}$ _____ o) $\sqrt[3]{1728}$ _____ s) $\sqrt[3]{1000}$ _____

d) $\sqrt[3]{512}$ _____ h) 12^3 _____ l) $\sqrt[3]{125}$ _____ p) $\sqrt[3]{4096}$ _____ t) $\sqrt[3]{729}$ _____

World 8-2 Missing Measures

1) Determine the missing side length of the following cubes given…

a) 

Total Area = 96 cm^2

b) 

Volume = 81 dm^3

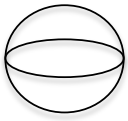
c) 

Base Perimeter = 10 cm

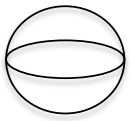
d) 

A_{Lat}= 324 mm^2

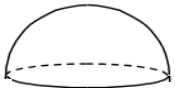
2) Determine the radius of each sphere or hemisphere given….

a) 

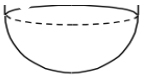
Total Area = 144π cm^2

b) 

Volume = 2304π m^3

c) 

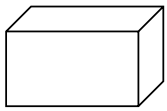
Volume is = 18π cm^3

d) 

A_T= 78.85 mm^2

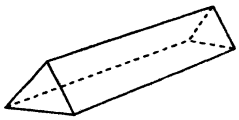
3) Calculate the missing height for each of the prisms

a)



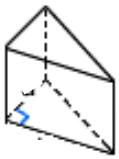
Volume = 400 m^3
l = 10 m, w = 16 m

b)



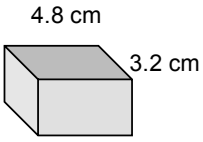
Triangle Perimeter = 15 cm
Triangle Area = 10.83 cm
Total Surface Area = 156.66 cm^2

c)



Triangle is a 3,4,5 right angled triangle
Volume is 90 dm^3

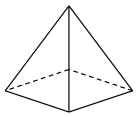
d)



Total surface area = 86.72 cm^2

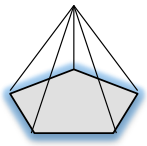
4) Calculate the missing measure for these pyramids and cylinders

a)



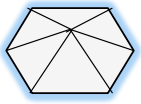
Square based pyramid
Volume is 176.4 m^3
Height is 10 m
Side length = ?

b)



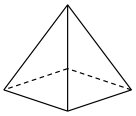
A pentagon based pyramid with base perimeter 25 m
Apothem is 3.44 m
Total area is 155.5 m^2
Slant length = ?

c)



Apothem =?
Height is 6 m
Volume is 32.55 m^3
Side length is 2.5 m

d)



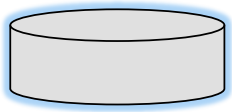
surface area: 312 m^2
Side length 12
Slant length = ?

5) Calculate the missing measure that is indicated in each question

a)

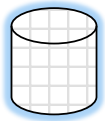
Volume is 384π dm^3
Radius is 6 dm
Height = ?

b)



Volume is 196 cm^3
Height = 16 cm
Radius = ?

c)



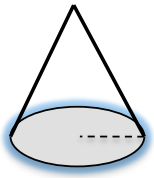
Volume is 346.4 m^3
Diameter is 7 m
Height= ?

d)

Height is 15 dm
Lateral Area is 3 dm
Volume = ?

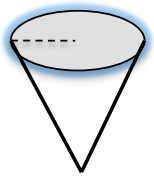
6) Calculate the missing measure that is indicated in each question

a)



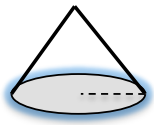
Volume is 133 m^3
Height is 7.2 m
Radius = ?

b)



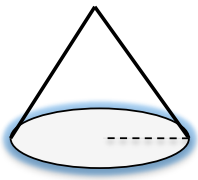
Volume is 420π mm^3
Height is 18 mm
Diameter = ?

c)



Lateral Area is 192π m^2
Slant Length is 12 m
Diameter = ?

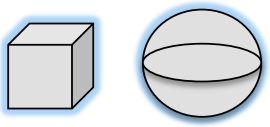
d)



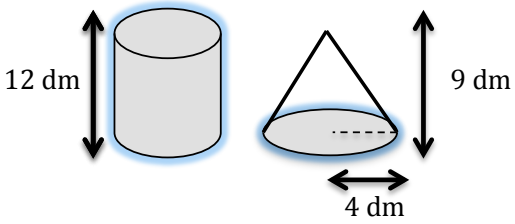
$A_{Lateral}$ is 1570 m^2
Radius is 25 m
Slant length = ?

World 8-3 Equivalent Solids

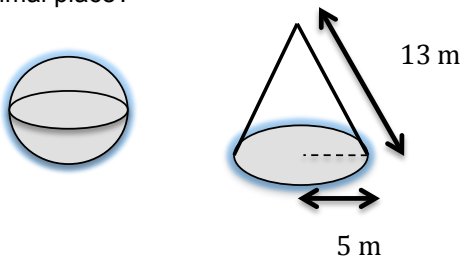
1) A cube and a sphere have the same volume. If the sphere's radius is 3 cm, what is the cube's total area?



2) A cylinder and a cone have the same volume. What is the total area of the cylinder?

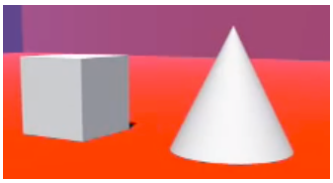


3).A sphere and a cone have the same total area. What is the radius of the sphere to 1 decimal place?



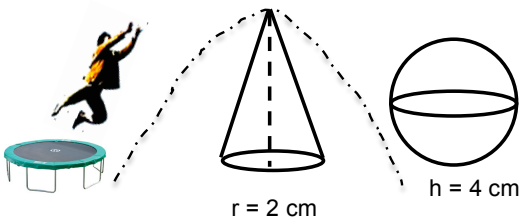
4) In LEVEL RED, Cube encounters the "Dreaded Cone." It turns out, they have **equal volumes.** The cube has a side length of 3.85 units. The cone has a height of 5 units As the programmer of Cube it is important to know two things.

a) The radius of the Dreaded Cone.
b) The surface area of the Dreaded Cone.

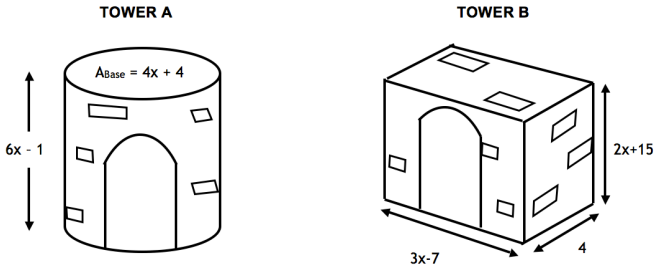


5) Jackie Chan can jump a **max height of 3.2 m**. He's going to jump over the *"danger cone."* If the sphere and the cone have the same total surface area will Jackie jump safely over the cone?

Jackie Danger Cone

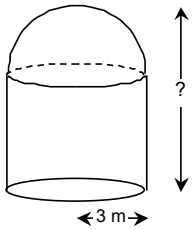


6) Star Dust the dragon is sitting on a pile of gold in tower A. Both treasure towers have the same volume. If the pile of gold reaches the ceiling in tower A, how tall is the pile of gold?

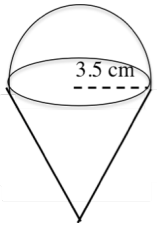


World 8-4 Missing Measures of Decomposable Solids

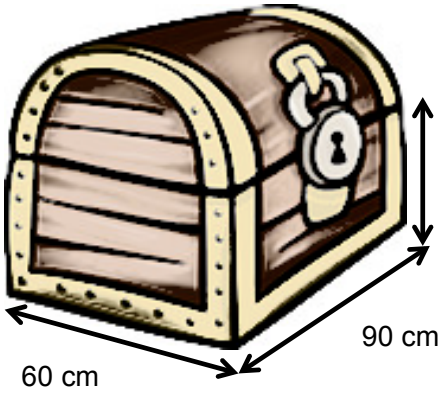
1. *Space Probe Mars 3* is heading towards the red planet. The total volume of the probe is 226 m^2 and the radius of its base measures 3 m.
What is the total height of *Space Probe Mars 3*?



2. An ice cream cone has a hemisphere on top of a cone with the same diameter.
What is the total height of the object if its total area is 198.158 cm^2 ?



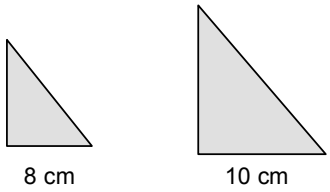
3. A treasure chest is made from half a cylinder and a rectangular based prism. If the total volume of the chest is 370 170 cm^3. What is the height?



World 8-5 Similar Figures: k factor

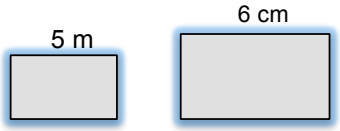
1. Determine the Scale Factor

a)



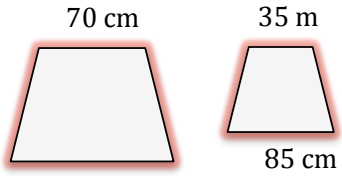
_____

b)



_____

c)



_____

2. True or False

a) Two square are always similar ____

b) Two rectangles are always similar ____

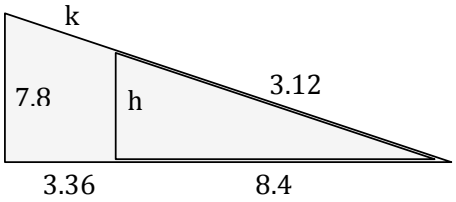
c) Two circles are always similar ____

d) Two isoseceles triangles are always similar ____

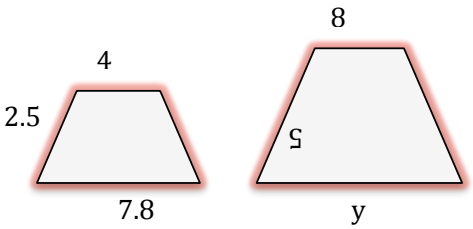
e) Two equilateral triangles are always similar ____

3. Determine the missing side lengths

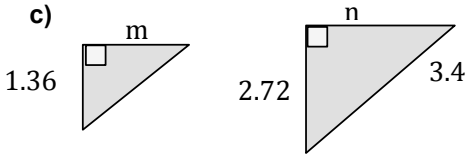
a)



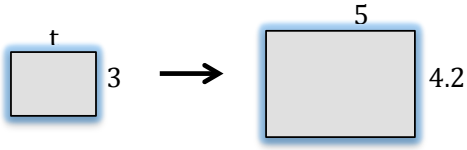
b)



c)



d)



4. Are the rectangles similar?

a) 10 cm by 8 cm: 30 cm by 24 cm

b) 42 km by 54 km: 56 km by 72 km

c) 18 m by 24 m: 27 m by 36 m

d) 15 cm by 48 cm: 10 cm by 30 cm

5. Are these triangles similar?

a) 5, 6 and 7 : 20, 24 and 21

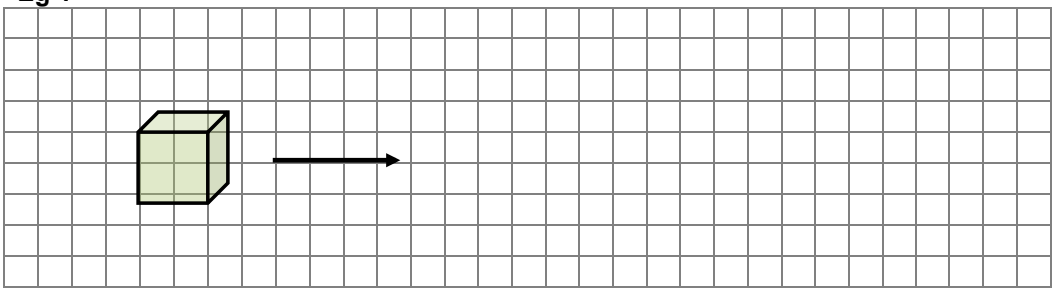
b) 8, 9, 14 : 48, 54 and 90

c) 18, 25, and 30 : 6, 8 and 10

World 8-6 Similar Solids

To create a similar solid, all you have to do is multiply ____ of the _____ dimensions of your original solid by a constant _____ factor (___).

Eg 1



What happened to the surface area and volume of the cube after being scaled?

Given that **length** increased by a ratio of ____

The **surface area** increased by a ratio of ____ or ____

The **volume** increased by a ratio of ____ or ____

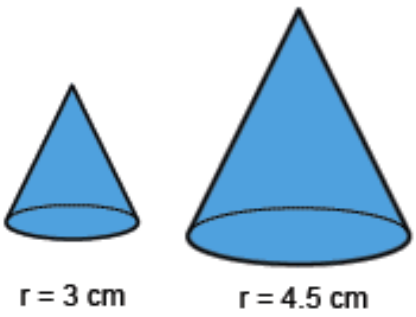
Therefore, when you increase the side length of solid by some scale factor.

The surface area increases by that factor _____ and the volume by that factor _____.

Length Ratio	Area Ratio	Volume Ratio

Eg 2 For the following similar cônes find;

a) the ratios of lengths,
b) the ratio of areas
c) the ratio of volumes

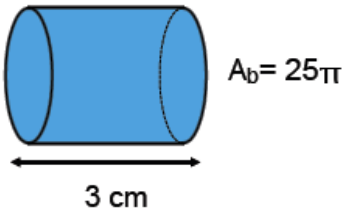


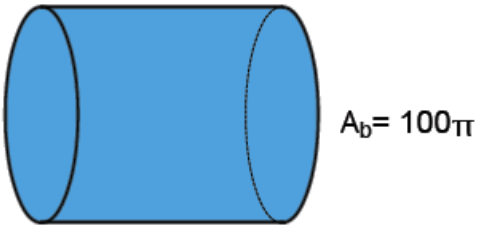
Eg. 3 Complete the following table

k	k^2	k^3
	4	
$\frac{3}{2}$		
	25	
	$\frac{16}{9}$	
		$\frac{27}{8}$
	$\frac{9}{25}$	
		729

KEY POINTS TO REMEMBER

Eg. 4 These two cylinders are similar. What is the volume of the larger one.





8-7 Similar Figures Practice Problems

PART A – MULTIPLE CHOICE

1) Prism B has a height 6 times that of Prism A and they are similar. **How many times greater is the volume of Prism B?**

A) *4*
B) *36*
C) *216*
D) 64

2) A photographer wants to enlarge some sports photos. She wants to enlarge a photo that is 5cm by 7cm so that the dimensions are three times larger than the original. **How many times larger will the area of the new photo be?**

A) *9 times*
B) *3 times*
C) *35 times*
D) 13 times

3) Triangle DEF and triangle ABC are similar.

The ratio of the areas of ΔABC: ΔDEF is 9 to 1, **what is the length of segment DE?**

A) *36 cm*
B) *5 cm*
C) *4 cm*
D) 1.33 *cm*

4) A sphere has a total volume of 2304p cm^3. What is the radius of the sphere?

A) *9 cm*
B) *12 cm*
C) *16 cm*
D) 14 *cm*

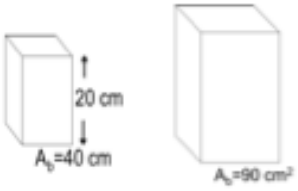
*5) A cone has a volume of 48*p m^3. If the radius is 2 m what is the height of this cone?

A) *3 cm*
B) *12 cm*
C) *16 cm*
D) 36 *cm*

PART B – SHORT ANSWER

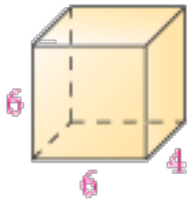
6) A cone shaped pool has a volume of 196π m^3 if the radius is 7 m what is the height of the pool?

7) A music store uses speakers that are in the shape of square based prisms. The two models used are similar. **What is the volume of the big speaker ?**



8) The solids below are similar.

a) What is the scale factor? _____
b) What is the ratio of their perimeters? _____
c) What is the ratio of the surface areas? _____
d) What is the ratio of their volumes? _____



9) There is a grown-up and a baby version to party hats. Determine the…..

Ratio of lengths _____ 5

Ratio of the total areas_____

Ratio of the total volumes _____

PART A – MULTIPLE CHOICE

1) Sphere B has a radius 4 times that of sphere A. **How many times greater is the volume of sphere B?**

A) *4*

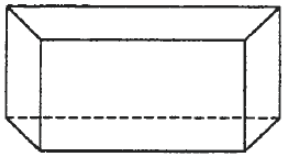
B) *16*

C) *8*

D) 64

2) Two trapezoidal based prisms are similar. The ratio of their heights is $\frac{3}{2}$. **What is the volume of the larger prism?**

Volume: 40 cm^3



Volume: ?

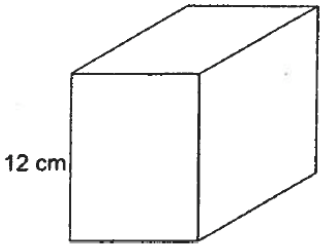
A) *60 cm^3*

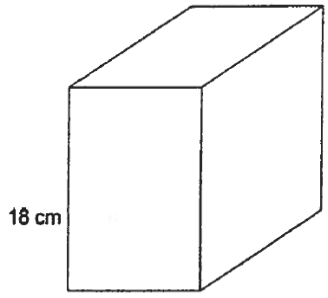
B) *180 cm^3*

C) *135 cm^3*

D) 90 *cm^3*

3). Two similar tissue boxes are shown below. Which of the following is **true?**





A) The length ratio is 6

B) *The area ratio is* $\frac{3}{2}$

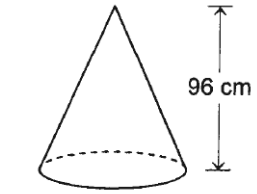
C) *The volume ratio is* $\frac{27}{8}$

D) The perimeter ratio is $\frac{9}{4}$

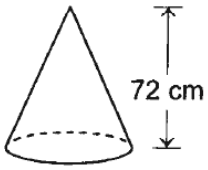
PART B – SHORT ANSWER

4) The area ratio of two cereal boxes is $\frac{64}{49}$. What is the ratio of their volumes?

5) Two right circular cones are similar. What is the lateral area of the smaller cone?



Lateral Area: $2\,800\,\pi$ cm^2



Lateral Area: ?

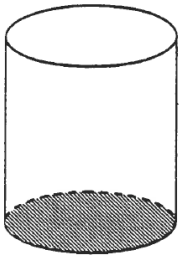
PART C – LONG ANSWER

6) The two cylinders are similar. What is the volume of the larger cylinder?

V = 72 cm^3

A_b = 16 cm^2

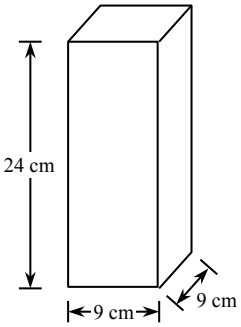
V = ?



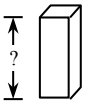
A_b = 144 cm^2

7) A candle maker melts down a large block of wax in the shape of a right prism with a square base to make candles that are similar to the large block of wax. Various measurements are given in the diagram below.

Block of wax that the candle maker melts



Candle produced by the candle maker



Volume: 72 cm^3

What is the height of each candle produced by the candle make?

Student Name

							Total
Criteria 1	0	8	16	24	32	40	
Criteria 2	0	8	16	24	32	40	
Criteria 3 & 4	0	4	8	12	16	20	

SITUATIONAL PROBLEM: #8

Summer Olympics

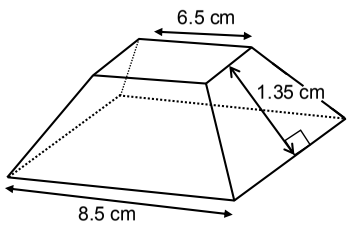
The next Summer Olympics are this August in London, England. You are the assistant to the purchasing director of the Olympic Committee.

- You must calculate the exact cost of purchasing all the medals (gold, silver, and bronze) for the events in the categories listed in the table below.
- You have been assigned a budget of $1 100 000.
- One gold, one silver, and one bronze medal are awarded at **each** event. There are no ties.

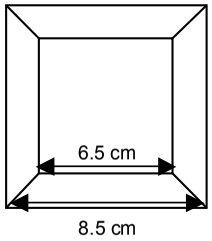
GOLD MEDALS

The solid chosen for the gold medals is a square-based right pyramid whose dimensions are provided below.

Three-dimensional (3-D) view



Top view

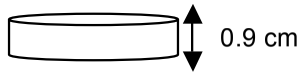


Because gold is very expensive, the medals will be made of another material and then coated in gold on all sides. The cost of the gold is $0.55/mm^2.

SILVER MEDALS

These medals are made entirely of silver, and are in the shape of a cylinder. Diagrams are not to scale.

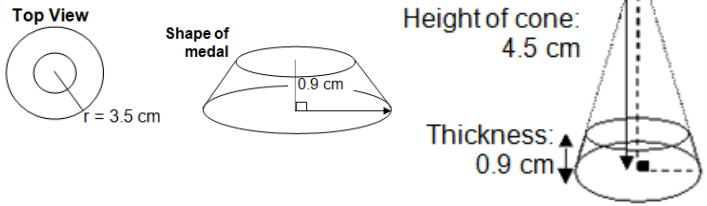
- Thickness of the medal: 0.9 cm
- Lateral area of the cylinder is 16.96 cm^2
- The cost of the silver is $12 / mL.



BRONZE MEDALS.

The bronze medals are made of solid bronze, and are in the shape of a cone with the top sliced off.

- Height of cone: 4.5 cm
- Radius of the base of the original cone: 3.5 cm
- Bronze, which is actually a mixture of copper and tin, costs $7.10 per cm^3.



The following table provides some information about the number of events in the top four categories.

Categories	Events in each Category
Athletics	47
Aquatic	(x+4)(x+5) +4
Equestrian	3x(4x – 5)
Gymnastic	$\frac{(2^3 \cdot 3^5)^3}{2^8 \cdot 3^{14}}$
Total	$13x^2 + 32x + 1$

Can you buy all of the medals with your given budget? How much money is left over or how much will you need?

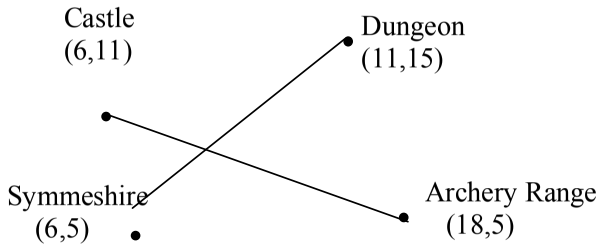
							Total
Criteria 1	0	8	16	24	32	40	
Criteria 2	0	8	16	24	32	40	
Criteria 3 & 4	0	4	8	12	16	20	

SITUATIONAL PROBLEM: #9
THE RED DRAGON

Legend speaks of a secret hideout for the king's most respected knight known as "The Red Dragon." As a peasant living in the medieval village of D'Arcyton you stumbled upon a long lost journal entry from The Red Dragon himself which includes a map, a poem and a diagram of his lair.

The Red Dragon's Journal
June 12, 1148 AD

Not the King nor the other knights know of my lair. I have placed a personal belonging here so that someday it can be found again. I've drawn a map to its general location. The location where the <u>two roads meet</u> is the location of my lair.



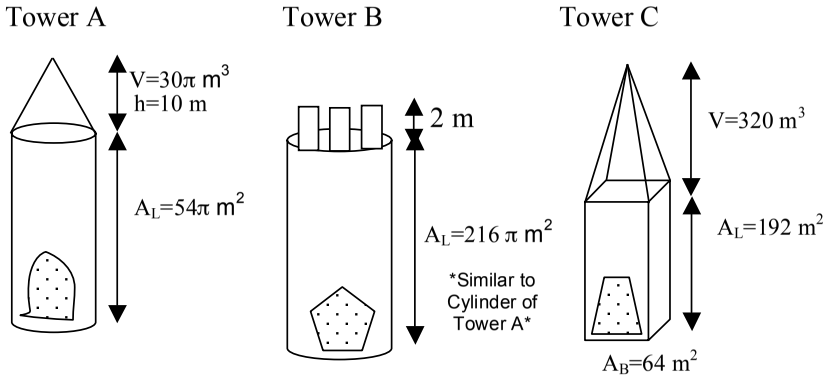
The Red Dragon's Poem
Take my helmet's, take my gold
Solve my riddle and then behold,
Take my helmets take my shields
wonder what my number yields?

$$(r-3)(r+4)+\frac{5(r^4d^3)^2}{(rd)^6}+\frac{15r^5+9r^4+12r^3}{3r^3}=\sqrt[3]{1331}r^2$$

Table 1 – Determining which Treasure is in the Lair

x and y are the coordinates of the dragon's lair *r is the solution from* the Dragon's Riddle	Hidden Treasure
If x < y & r = 2	Red Dragon Shield
If x < y & r = 3	Red Dragon Helmet
Anything Else	Gold

The Red Dragon's Towers Diagram
The tallest tower holds the hidden treasure.
Note: Diagrams are not to scale



- **Tower A** is composed of a cone and a cylinder.
- **Tower B** is a 2 m tall guard rail on top of a cylinder. The cylinder is <u>similar</u> to the cylinder in Tower A.
- **Tower C** is a decomposable square based pyramid on top of a square based prism.

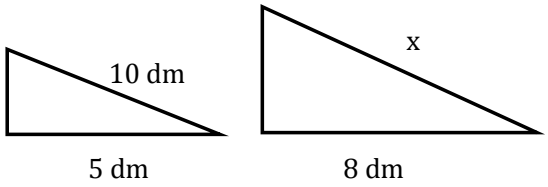
1] What is the location of the Red Dragon's Lair?
2] Which treasure is hidden in the Red Dragon's Lair?
3] What tower is it located in?

Practice Test #5
Similar Figures and Missing Measures

Name: _____ Date: _____

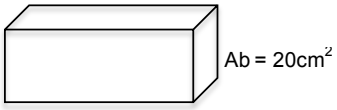
PART A: Multiple Choice *write the correct letter in the space provided (2 marks each)*

1) These two triangles are similar. What is the length of side x?



a) 13 dm b) 10 dm c) 16 dm d) 25.6 dm

2) A prism has a total volume of 420 cm^2. Determine the height.

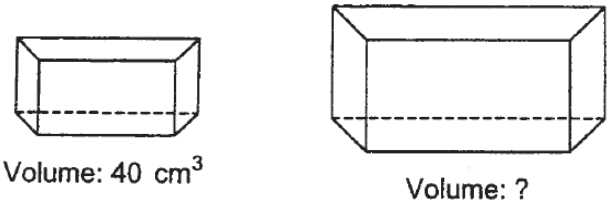


a) 2 cm b) 21 cm c) 4.6 cm d) 10.5 cm

3) If the surface area ratio of two similar cylinders is 64 what is the ratio of their lengths?

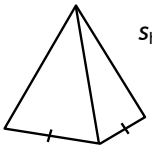
a) 5 b) 6 c) 7 d) 8

4) Two trapezoid based prisms are similar. The ratio of their heights is 1.5. What is the volume of the larger prism?



a) 60 cm^3 b) 135 cm^3 c) 180 cm^3 d) 90 cm^3

5) Two square based pyramids are similar. What is the lateral area of the smaller pyramid?



Lateral Area = 360 m^2

Lateral Area = ? m

a) 540 m^2 b) 360 m^2 c) 250 m^2 d) 160 m^2

6) A sphere has a total volume of 2304π cm^3. What is the radius of the sphere?

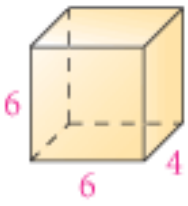
a) 9 cm b) 12 cm c) 18 cm d) 14 cm

PART B: Short Answer *write the correct letter in the space provided*

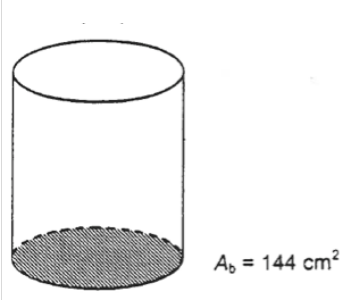
7) A cone has a total volume of 268.1 cm^3 and a radius of 4 cm. What is the height of the cone? h _____ cm.

8) The solids below are similar.

a) What is the scale factor? _____
b) What is the ratio of their perimeters? _____
c) What is the ratio of the surface areas? _____
d) What is the ratio of their volumes? _____



9) The two cylinders are similar. What is the height of the larger cylinder? h =_____ cm



LONG ANSWER *Show all of your work. Include a final statement. (30 marks)*

10. PYRAMID UNDER THE SAND

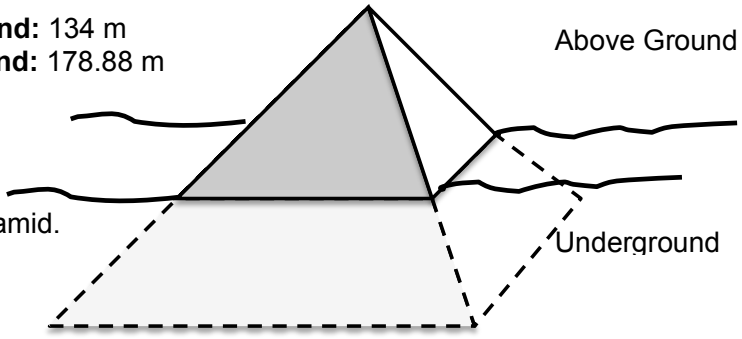
A 5000 year old square based pyramid has been discovered recently in Egypt. A large portion of it has been covered by sand and is now underground. A group of archaeologists have uncovered some important information about this pyramid.

They want to know what percent of the pyramid's volume remained above the ground.

Pyramid Height Above Ground: 134 m
Pyramid Slant Height Above Ground: 178.88 m

Total Height of Pyramid: 201 m

Note: The portion of the pyramid above ground is similar to whole pyramid.



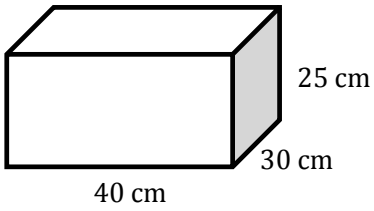
_____ % of the volume of the pyramid is above ground.

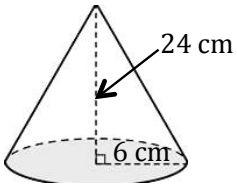
Uses mathematical reasoning							
		Observable indicators correspond to level					
Evaluation Criteria	LEVEL	A	B	C	D	E	
	Cr. 3	40	32	24	16	8	0
	Cr. 2	40	32	24	16	8	0
	Cr. 4 Cr. 5	20	16	12	8	4	0

11. FISH FOOD

Carson's aquarium, in the shape of a rectangular prism, is filled to 80% of its height. Next, he dropped ***three*** solid food cones into the tank.

The dimensions of the tank are 40 cm long, by 30 cm wide, by 25 cm high. The food cone has a radius of 9 cm and a height of 24 cm. (Diagrams are below… not to scale!)





Does the tank overflow? If so, how much water, in litres, spills over? If not, how much room is left in the tank, in litres?

Does the tank spill over? **Yes** **No** (circle one)

How much spills over, or how much room is left?

_____ L

Uses mathematical reasoning							
		Observable indicators correspond to level					
Evaluation Criteria	LEVEL	A	B	C	D	E	
	Cr. 3	40	32	24	16	8	0
	Cr. 2	40	32	24	16	8	0
	Cr. 4 Cr. 5	20	16	12	8	4	0