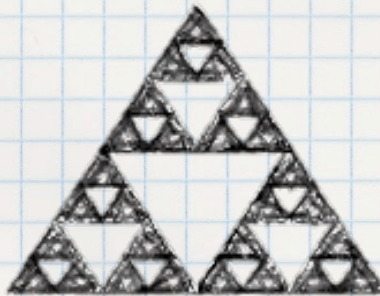
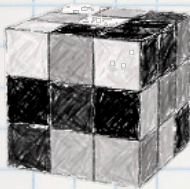


$$A_T = 2\pi r^2 + 2\pi r h$$

# MATH STUDENT'S GUIDE TO THE GALAXY



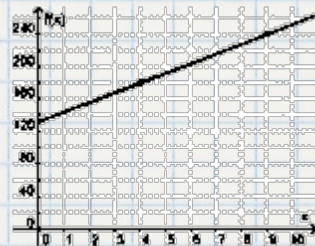
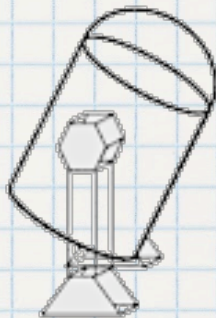
$$b^2 = c^2 - a^2$$



Math 306 Edition

2014-2015

$$(3x+5)(2x-5) = -8$$



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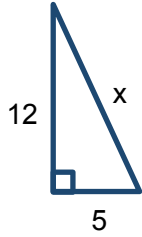
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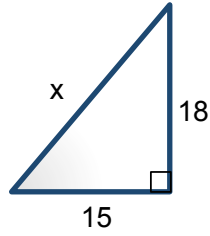
### World 1-1 Pythagoras' Theorem

1) Determine the missing side length using Pythagoras' Theorem

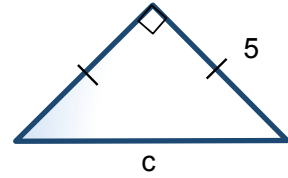
a)



b)

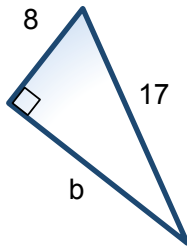


c)

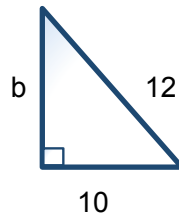


2) Determine the missing side length

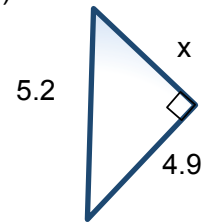
a)



b)



c)



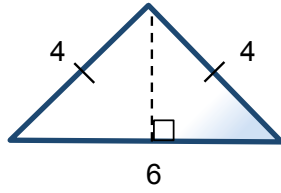
3) Do the following set of 3 side lengths of a triangle make a right-angled triangle?

a) 7.5 cm, 10 cm, & 12.5 cm

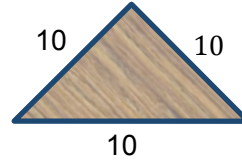
b) 12 m, 15 m & 21 m

4) Calculate the area of these triangles

a)



b)

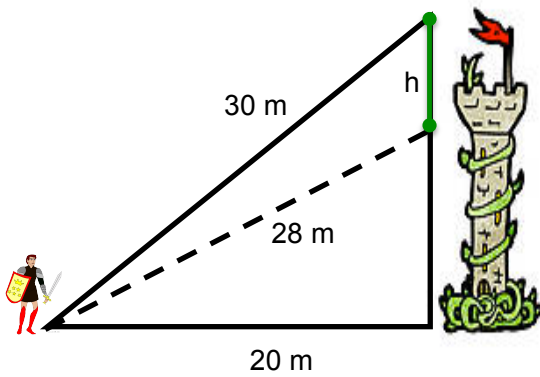


5) Complete the two tables of Pythagorean triples

a	b	c
3	4	
6		10
	12	13

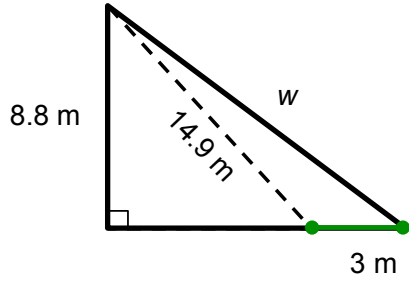
a	b	c
1.5	2	
	24	25
$\sqrt{55}$	$\sqrt{89}$	

6) Determine the length of segment h



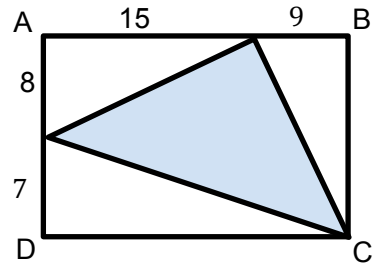
### World 1-2 Pythagoras Theorem Continued

1) Calculate the length of  $w$

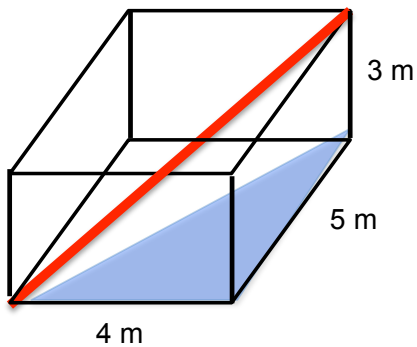


2) a) Calculate the perimeter of triangle ABC

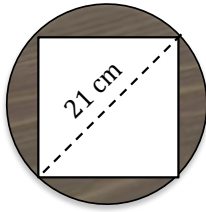
b) Calculate the area of the shaded triangle



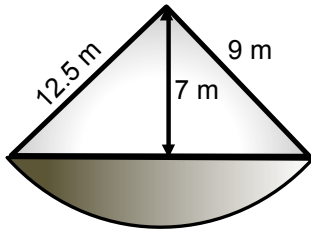
3) Determine the length of a string hung from one corner of the room to the other.



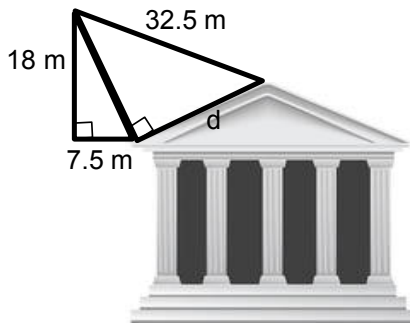
4) A cross section of a log has been cut. Determine the area of the shaded part of the log.



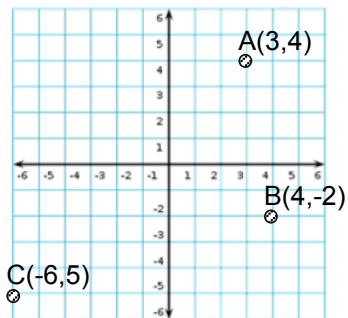
5) Three of the dimensions of a sailboat's mast are shown below. How wide is the boat?



6) What is the slant length of the roof of the Parthenon?



7) Determine the distances from the origin (0,0) to points A, B and C on the Cartesian plane to 1 decimal place      Distance to A \_\_\_\_\_ B \_\_\_\_\_ C \_\_\_\_\_ from origin.



## World 1-3 Basic Exponent Laws

### Introduction to Powers

1) Write the following products using exponential notation. Do not solve. (ex:  $2^3 = 2 \times 2$ )

a)  $7 \times 7 \times 7 \times 7 =$  \_\_\_\_\_      b)  $3 \times 3 \times 3 \times 3 \times 3 =$  \_\_\_\_\_      c)  $-2 \times -2 \times -2 =$  \_\_\_\_\_  
 d)  $b \times b =$  \_\_\_\_\_      e)  $-a \times a \times a \times a \times a =$  \_\_\_\_\_      f)  $\frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} =$  \_\_\_\_\_

2) Perform the following calculations

a)  $3^3 =$  \_\_\_\_\_      b)  $(-2)^5 =$  \_\_\_\_\_      c)  $(1.8)^1 =$  \_\_\_\_\_  
 d)  $(\sqrt{10})^2 =$  \_\_\_\_\_      e)  $256^0 =$  \_\_\_\_\_      f)  $\left(\frac{2}{3}\right)^3 =$  \_\_\_\_\_

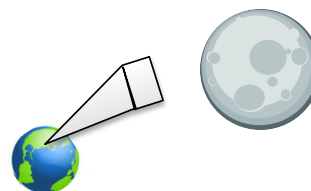
3) Calculate

a)  $3^4 \times 3^2 =$  \_\_\_\_\_      b)  $3^4 + 3^2 =$  \_\_\_\_\_      c)  $3^4 \div 3^2 =$  \_\_\_\_\_  
 d)  $(3^2)^4 =$  \_\_\_\_\_      e)  $2^4 \times 3^2 =$  \_\_\_\_\_      f)  $\frac{3^4}{2^3} =$  \_\_\_\_\_

### Multiplying Powers

1) Fill in the following table

Question	Expanded Form	Single Exponent
$3^2 \times 3^1$	$3 \times 3 \times 3$	$3^3$
$5^3 \times 5^2$		
$4^4 \times 4^3$		
$a^m \times a^n$		



General Rule:

2) Write as a single exponent:

a)  $8^4 \times 8^4 =$  \_\_\_\_\_      b)  $5^3 \times 5^7 =$  \_\_\_\_\_      c)  $2^2 \times 2^5 \times 2^3 =$  \_\_\_\_\_  
 d)  $(4^0)(4^3)(4^1) =$  \_\_\_\_\_      e)  $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^4 =$  \_\_\_\_\_      f)  $(-5)^4 \cdot (-5) =$  \_\_\_\_\_



3) Write as a single exponent:

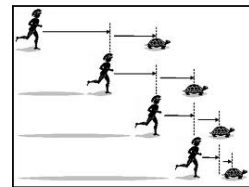
a)  $(x^2)(x^3) = \underline{\hspace{2cm}}$       b)  $y \cdot y^4 = \underline{\hspace{2cm}}$       c)  $(a^2)(a^0)(a^5) = \underline{\hspace{2cm}}$   
 d)  $a^2b^2c^3b^3 = \underline{\hspace{2cm}}$       e)  $m^4n^3 \cdot m^2n^5 = \underline{\hspace{2cm}}$       f)  $(3x^2)(3x^4)(3x^7) = \underline{\hspace{2cm}}$

4) Simplify

a)  $3a^4 \cdot 2a^2 = \underline{\hspace{2cm}}$       b)  $(-x^3)(x^7) = \underline{\hspace{2cm}}$       c)  $(8ab^2)(2a^2b^3) = \underline{\hspace{2cm}}$   
 d)  $\left(\frac{3}{2}x^5y^2\right)\left(\frac{2}{3}xy^3\right) = \underline{\hspace{2cm}}$       e)  $\left(\frac{3}{4}abc\right)\left(\frac{2}{5}abc\right) = \underline{\hspace{2cm}}$       f)  $(-3a^5b^7)(-5a^3b^{10}) = \underline{\hspace{2cm}}$

**Dividing Powers**    1) Fill in the following table

Question	Expanded Form	Single Exponent
$2^4 \div 2^4$	$\frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2}$	$2^0$
$3^5 \div 3^2$		
$5^7 \times 5^3$		
$a^m \times a^n$		



General Rule:

2) Write as a single exponent:

a)  $\frac{8^7}{8^4} = \underline{\hspace{2cm}}$       b)  $\frac{2^2}{2^4} = \underline{\hspace{2cm}}$       c)  $\frac{(-4)^{10}}{(-4)^5} = \underline{\hspace{2cm}}$   
 d)  $\frac{a^5}{a^4} = \underline{\hspace{2cm}}$       e)  $\frac{x^{10}y^{12}}{x^3y^9} = \underline{\hspace{2cm}}$       f)  $\frac{18m^8}{6m} = \underline{\hspace{2cm}}$

3) Simplify using exponent rules:

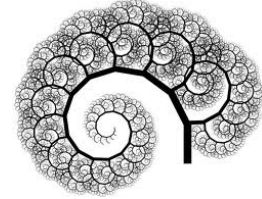
a)  $\frac{12x^{12}}{16x^5} = \underline{\hspace{2cm}}$       b)  $\frac{(x^7)(x^3)(x^5)}{(x^4)(x^{10})} = \underline{\hspace{2cm}}$       c)  $\frac{4a^6}{b} \cdot \frac{b^6}{16a^3} = \underline{\hspace{2cm}}$   
 d)  $\frac{x^{11}b^7}{x^4b^2} = \underline{\hspace{2cm}}$       e)  $\frac{a^0b^0c^0}{b^0c^0a^0} = \underline{\hspace{2cm}}$       f)  $\frac{3x^5}{4y^3} \cdot \frac{y^8}{9x^{12}} \cdot \frac{12x^7}{y^2} = \underline{\hspace{2cm}}$

## World 1-4 Special Exponent Laws

### Power of A Power

1) Fill in the following table

Question	Expanded Form	Single Exponent
$(3^2)^4$	$(3^2)(3^2)(3^2)(3^2)$	$3^8$
$(5^3)^3$		
$(x^3)^2$		



General Rule:

2) Calculate:

a)  $(2)^3 \times (2)^2 = \underline{\hspace{2cm}}$       b)  $(1^3)^4 = \underline{\hspace{2cm}}$       c)  $((-1)^3)^4 = \underline{\hspace{2cm}}$

d)  $(3)^2 \times (4)^2 = \underline{\hspace{2cm}}$       e)  $(-3^2)^3 = \underline{\hspace{2cm}}$       f)  $((-3)^2)^3 = \underline{\hspace{2cm}}$

3) Simplify using exponent rules:

a)  $(a^4)^3 = \underline{\hspace{2cm}}$       b)  $(x^2)^5 \cdot (x^6)^3 = \underline{\hspace{2cm}}$       c)  $(m^3)^0 = \underline{\hspace{2cm}}$

d)  $(y^8)^2 \cdot y^4 \cdot (y^3)^6 = \underline{\hspace{2cm}}$       e)  $(-z)^3 = \underline{\hspace{2cm}}$       f)  $\frac{(w^8)^3}{(w^6)^4} = \underline{\hspace{2cm}}$

### Power of A Product

1) Fill in the following table

Question	Using BEDMAS	Using Distributive Law of Exponents
$(3 \times 2)^2$	$(6)^2 = 36$	$3^2 \times 2^2 = 9 \times 4 = 36$
$(2 \times 6)^3$		
$(4 \times 5)^2$		

2) Simplify using exponent rules:

a)  $(2a)^3 = \underline{\hspace{2cm}}$       b)  $(3a^2)^4 = \underline{\hspace{2cm}}$       c)  $(x^2y^5)^2 = \underline{\hspace{2cm}}$

d)  $(2^3x^5y)^6 = \underline{\hspace{2cm}}$       e)  $(-4m^2n)^3 = \underline{\hspace{2cm}}$       f)  $[(a^2b)^4]^3 = \underline{\hspace{2cm}}$

3) Simplify using exponent rules:

a)  $(\frac{2}{3}z^2)^3 =$       b)  $(2x^3)^4 (3x^2)^4 =$

c)  $(2n^4)^2(n)^4$       d)  $(-2y^3z^4)^4(3y^5z^3)^2$

e)  $(\frac{1}{2}p^3q^4)^2 (4pq^2)^3$       f)  $(3a^5b)^2 \cdot (a^2b)^5 \cdot (a^2b)^5$

**Power of a Quotient**

1) Fill in the following table

Question	Using BEDMAS	Using Distributive Law of Exponents
$(\frac{15}{3})^2$	$(5)^2 = 25$	$\frac{15^2}{3^2} = \frac{225}{9} = 25$
$(\frac{12}{4})^3$		
$(\frac{25}{5})^2$		

2) Simplify using exponent rules:

a)  $(\frac{a}{b})^4 = \underline{\hspace{2cm}}$       b)  $(\frac{2x}{3y})^2 = \underline{\hspace{2cm}}$       c)  $(\frac{4m}{n})^2 = \underline{\hspace{2cm}}$

d)  $(\frac{x^2}{2y^3})^3$       e)  $(\frac{2m^5}{5n^4})^2 (\frac{m^5}{2n^4})^3$       f)  $(\frac{-2a^2}{b})^3 (\frac{b^2}{3a})^2$

3) Simplify using exponent rules:

a)  $\left(\frac{x^4y^8}{x^3y^5}\right)^2 =$

b)  $\frac{20m^8n^4}{25m^4n} =$

c)  $\left(\frac{4a^6b^4}{2a^3}\right)^3 =$

d)  $(5x^2y)(2xy^2) =$

e)  $\left(\frac{p^6}{q^4}\right)^4 \left(\frac{q^6}{p^8}\right)^3 =$

f)  $\left(\frac{2a^3}{5b^4}\right)^3 \div \left(\frac{a^4}{5b^3}\right)^2 =$

**Negative and Fractional Exponents**

1) Evaluate using your calculator:

a)  $4^{1/2} =$  \_\_\_\_\_

b)  $8^{1/3} =$  \_\_\_\_\_

c)  $100^{1/2} =$  \_\_\_\_\_

d)  $729^{1/3} =$  \_\_\_\_\_

e)  $(-125)^{1/3} =$  \_\_\_\_\_

f)  $15625^{1/2} =$  \_\_\_\_\_

2) Evaluate. Leave your answer as a fraction or integer. No decimals

a)  $2^{-1} =$  \_\_\_\_\_

b)  $\left(\frac{1}{4}\right)^{-1} =$  \_\_\_\_\_

c)  $10^{-1} =$  \_\_\_\_\_

d)  $\left(\frac{1}{3^2}\right)^{-1} =$  \_\_\_\_\_

e)  $\left(\frac{1}{5}\right)^{-2} =$  \_\_\_\_\_

f)  $\left(\frac{3}{2}\right)^{-2} =$  \_\_\_\_\_

3) Simplify. Leave your answers with positive exponents.

a)  $x^{-1} =$  \_\_\_\_\_

b)  $\left(\frac{x}{y}\right)^{-1} =$  \_\_\_\_\_

c)  $\frac{n^{10}}{n^{12}} =$  \_\_\_\_\_

d)  $\frac{a^5b^7}{a^8b^4} =$  \_\_\_\_\_

e)  $(x^{-5})^{-3}(x^{-6})^2 =$  \_\_\_\_\_

f)  $(x^2y^4)^3(x^5y)^{-2} =$  \_\_\_\_\_

**Additional Practice: Use a Scrap piece of Paper to perform calculations**

1) Simplify using exponent rules:

a) $(3^{-2})(3^4)(3^6) =$ _____	b) $(n^7)(n^2)(n^{-4}) =$ _____	c) $(x^4)^3 =$ _____
d) $(3x^2)(2x^5) =$ _____	e) $\left(\frac{3}{x}\right)^{-2} =$ _____	f) $\left(\frac{a^4b^2c^5}{a^2b^6c^4}\right)^0 =$ _____
g) $(-2m^4n^3o)^3 =$ _____	h) $\left(\frac{3y^4}{5y^3}\right)^2 =$ _____	i) $\left(\frac{a}{b}\right)^2 \left(\frac{b}{c}\right)^4 \left(\frac{c}{a}\right)^3 =$ _____
j) $\left(\frac{p^4q^3}{p^2q^3}\right)^5 =$ _____	k) $\frac{36x^7y^8z^{12}}{6x^9y^5z^5} =$ _____	l) $(x^2y^3x^{-4}) \div (xy^{-2}z^3) =$ _____

### World 1-5 Scientific Notation

1) a) Large Object: Complete the table using scientific notation

Object	Decimal Notation Diameter in kilometers	Scientific Notation Diameter in kilometers
Earth	12 756	
Saturn	116 464	
Jupiter	142 984	
Sun	1 392 000	
Aldebaran	59 770 000	
Betelgeuse	903 500 000	
VV Cephei	2 644 800 000	



b) Using your own words, explain how you can turn a **positive** number from decimal notation to scientific notation. Be specific.

2) a) Tiny Objects: Complete the table using scientific notation

Object	Decimal Notation Size in meters	Scientific Notation Size in meters
Salt Grain	0.005	
Skin Cell	0.000 03	
Chromosome	0.000 007	
HIV Virus	0.000 000 130	
Hepatitis B	0.000 000 045	
Water Atom	0.000 000 000 275	



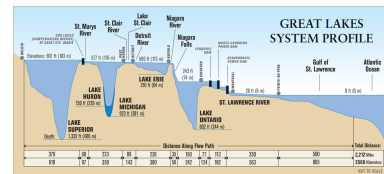
b) Using your own words, explain how you can turn a **negative** number from decimal notation to scientific notation. Be specific.

3) Calculate in decimal notation

- a)  $5.76 \times 10^4 =$  \_\_\_\_\_      b)  $0.315 \times 10^6 =$  \_\_\_\_\_  
 c)  $34.79 \times 10^3 =$  \_\_\_\_\_      d)  $5.66 \times 10^{-2} =$  \_\_\_\_\_  
 e)  $8.95 \times 10^6 =$  \_\_\_\_\_      f)  $4.5 \times 10\ 000 =$  \_\_\_\_\_  
 g)  $20.95 \times 0.001 =$  \_\_\_\_\_      h)  $31.4 \times 1000 =$  \_\_\_\_\_

4) Write, in decimal notation, the surface area of the Great Lakes.

- Lake Superior       $8.27 \times 10^4 \text{ km}^2$  \_\_\_\_\_  
 Lake Michigan       $5.83 \times 10^4 \text{ km}^2$  \_\_\_\_\_  
 Lake Huron       $6.16 \times 10^4 \text{ km}^2$  \_\_\_\_\_  
 Lake Erie       $2.50 \times 10^4 \text{ km}^2$  \_\_\_\_\_  
 Lake Ontario       $1.95 \times 10^4 \text{ km}^2$  \_\_\_\_\_



5) Express the **total** surface area of the Great Lakes in scientific notation.

\_\_\_\_\_

6) Express the following in scientific notation.

- a) Canada's federal debt is roughly \$590 464 000 000. \_\_\_\_\_  
 b) The diameter of a human cell is 0.000 000 11 m \_\_\_\_\_  
 c) The mass of a water droplet is 0.000 000 052 kg \_\_\_\_\_

7) Write in either scientific or decimal notation

- a) 4 500 = \_\_\_\_\_      b)  $2.4 \times 10^4 =$  \_\_\_\_\_  
 b) 0.005785 = \_\_\_\_\_      d)  $1.687 \times 10^{-8} =$  \_\_\_\_\_  
 c) 47 300 = \_\_\_\_\_      e)  $2.39 \times 10^4 =$  \_\_\_\_\_  
 d) 0.000 08 = \_\_\_\_\_      d)  $5 \times 10^{-2} =$  \_\_\_\_\_  
 e) 5 587 000 = \_\_\_\_\_      e)  $9.85 \times 10^1 =$  \_\_\_\_\_

8) Calculate and express the answer in scientific notation:

a)  $(5 \times 10^3) \cdot (3.5 \times 10^2) =$  \_\_\_\_\_

b)  $(5.5 \times 10^{-4}) \cdot (7.25 \times 10^6) =$  \_\_\_\_\_

c)  $(-4.25 \times 10^3) \cdot (3.5 \times 10^{-4}) =$  \_\_\_\_\_

d)  $(9 \times 10^3) \div (3.5 \times 10^2) =$  \_\_\_\_\_

9) Calculate and write the final answer using scientific notation.

a)  $\frac{(1.4 \times 10^4)(6.3 \times 10^6)}{(5.6 \times 10^8)} =$

b)  $\frac{(8.9 \times 10^9)(6.2 \times 10^{-3})}{(3.1 \times 10^2)} =$

c)  $\frac{(1.4 \times 10^4)(7.3 \times 10^{-4})}{(5.6 \times 10^{-4})} =$

d)  $\frac{(1.4 \times 10^4)}{(5.6 \times 10^8)(7.4 \times 10^{12})} =$

#### Memory Aid Space

Use this area to summarize **ALL** of the exponent laws, scientific notation and Pythagoras theorem. Include examples to illustrate your understanding.

Student Name \_\_\_\_\_

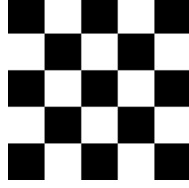
	Total					
Criteria 1	0	8	16	24	32	40
Criteria 2	0	8	16	24	32	40
Criteria 3 & 4	0	4	8	12	16	20

### MINI SITUATIONAL PROBLEM #1: A GIFT FROM THE QUEEN

Legend has it that in ancient India, a farmer saved a Queen from kidnapers. In return the Queen offered the farmer one wish.

The farmer humbly accepted the Queen's offer but gave her three options of repayment in rice to feed his village.

**Option 1** – If the Queen placed one grain of rice on the first tile and doubled that number on the second tile, then doubled that new number on the third tile, he'd like the number of grains of rice on the 25<sup>th</sup> tile.



**Option 2** – He would like a grain of rice for the total number of seconds there are in 1 year.

**Option 3** – He would like the number of grains of rice corresponding to the numerical value of  $\frac{(x^2)^5(x^2)^3}{(x^3)^3}$ . He suggested to her, she could find the value of x first, from the equation.  $6x - 12 = 4x + 10$ .

**Which option gives the farmer the most grains of rice? Provide your answers for each option in scientific notation.**